FRACTAL EPISODICITY OF GREAT EXPLOSIVE ERUPTIONS ON KAMCHATKA DURING HOLOCENE

A.A Gusev[1], V.V. Ponomareva[1], O.A.Braitseva[1], I.V.Melekestsev[1], L.D.Sulerzhitsky[2]

[1]Institute of Volcanic Geology and Geochemistry, Petropavlovsk-Kamchatsky, Russia; *gusev@emsd.iks.ru*
[2]Geological Institute, Pyzhevsky per., 7, Moscow, 109017, Russia

Introduction

Is the temporal structure of the sequence of volcanic eruptions uniform/Poissonian, clustered/episodic or cyclic/periodic? This question is interesting in itself, and crucial for understanding the impact of volcanoes on human habitat and climate. Over the geological time scale, the temporal character of volcanism is known to be non-uniform, *episodic* for such processes as ocean ridge volcanism, hot spot volcanism, explosive volcanism in island arcs (Sigurdsson, 2000), and trap (areal basalt) volcanism (Makarenko 1982). With respect to island arcs, this episodic style was established, on the basis of the study of volcanic ash layers in ocean-bottom boreholes (Kennet and others. 1977; Rea and Scheidegger 1979: Cambray and Cadet, 1996; Prueher and Rea, 2001). However, these studies presented only qualitative analysis. More formal description for the episodic temporal structure of volcanic (or, rather intrusive) processes was proposed was recently suggested by Pelletier (1999). He found that the actual episodicity in formation of intrusions is far from being completely "wild": when treated as random objects, intrusions are distributed in time (moreover, in space-time) in a statistically self-similar manner. The results of Pelletier confirm the general idea of an episodic behavior, and suggest that the episodicity of volcanic process may be generally (or typically) self-similar. For historic time scales, indications of a statistically self-similar or fractal behavior was found by DuBois and Cheminee (1991) for eruptions of basaltic volcanoes; and by Godano and Civetta (1996) for Etna; similar style was deduced by Chouet and Shaw (1991) for the burst-like behavior of a developing eruption. On the other side, many studies (e.g. Wickman, 1966; Ho and others, 1991; Jones and others, 1999) either assume or try to prove that eruptions of a particular volcanic center or of an area behave in another way: purely randomly (as a Poisson process) or with periodic tendency, and not as episodes.

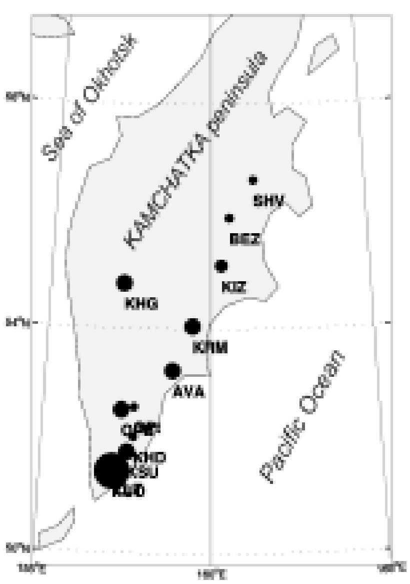

Figure 1. A scheme of 16 eruptive centers. The size of a dot indicates the relative size of the largest Holocene eruption at the respective center.

Data and eruption size distribution

We analyze the temporal structure of the sequence of largest (volume $V \geq 0.5$ km^3) explosive eruptions on Kamchatka in Holocene (for the last 10000 years) (Gusev and others, 2003). Our catalogue includes 29 events; most of them were revealed and investigated during long-term tephrochronological studies and dated by the radiocarbon method. For each of 16 eruptive centers, the relative size of maximum eruption is shown in Figure 1.The catalog it is presumably complete for eruptions with volumes of products exceeding 0.6-0.8 km^3. The high level of completeness is confirmed by the good fit of the corresponding size distribution by the hyperbolic/Pareto law, with the value of the exponent $b = 0.65$ (Fig. 2).

Time sequence and its renewal model

The temporal structure of the catalogue is seen in Figure 3. A visual inspection of the temporal sequence makes an impression that the moments of eruptions form tight groups or clusters. To verify this apparent tendency in a formal manner, we depart from the renewal process model. For this purpose we approximated the distribution of intervals Δt between the successive events by the Weibull law. On Figure 4A, the empirical cumulative distribution function $P(\Delta t' > \Delta t)$ is plotted on the Weibull law

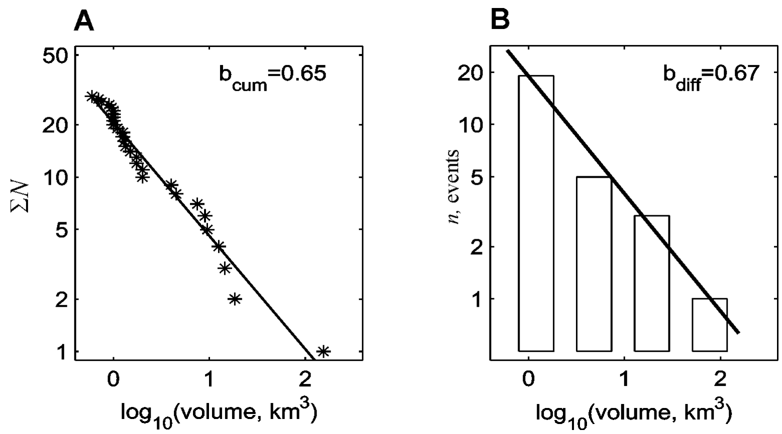

Figure 2. A: Non-normalized complementary cumulative distribution function of volumes of products of eruptions (the number of events with volume $V' > V$ vs. V). B: Same data represented as a histogram. Approximations by straight lines are good, indicating that the assumption of hyperbolic/Pareto distribution law is acceptable.

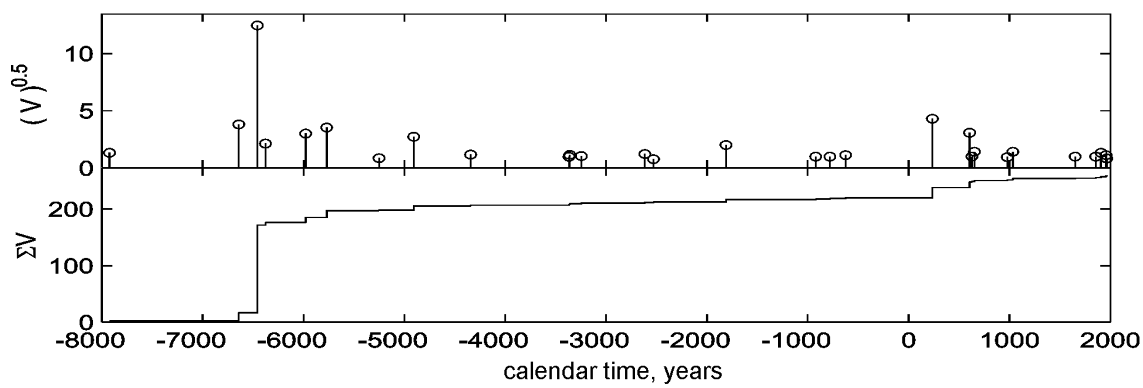

Figure 3. The temporal sequence of Holocene explosive eruptions on Kamchatka. Above: volumes V of eruptions, km^3. For visual clarity, the values of $V^{0.5}$ are plotted. Below: same data shown as the total volume of products accumulated up to a particular date.

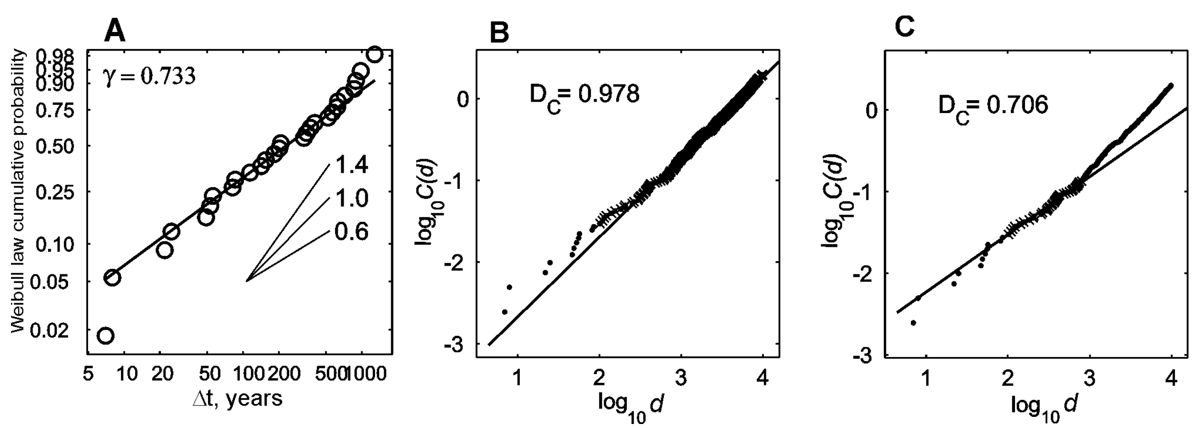

Figure 4. Plots illustrating clustering for the dates of eruptions. A: delays between successive events shown as a cumulative plot on the Weibull law probability paper. B: correlation integral as a function of an inter-event delay d, yr; the range of delays 100-10000 years is used for determination of the slope giving the estimate of correlation dimension near to unity. C: same as B, but for the range of delays 100-800 years; band-limited fractal behavior is seen.

probability paper. The graph is nearly linear, indicating that the Weibull law hypothesis is applicable. The slope of the straight line yields an estimate for the parameter of the Weibull law $\gamma = 0.73 < 1$, indicating the tendency of dates to clustering. The null hypothesis $\gamma = 1$ (that specifies Poissonian, "purely random" process) is rejected at the level of statistical significance equal to Q =7 %. With as few as 28 intervals, we consider this result as an indication of real clustering. We are mostly interested whether the clustering has the self-similar features. Hence, we further checked the presence of such features employing three independent techniques.

Correlation integral approach

The first approach used was to estimate the correlation dimension D_c of the set of points (that is, of dates of events on the time axis) by means of empirical determination of correlation integral

$$C(d) = (1/N_p)\, N(d_{ij} < d)$$

where N_p is the total number of possible pairs of points on time axis, $d_{ij}=t_j-t_i$ is the delay between components of a pair with the numbers i and j ($j > i$), and $N(d_{ij}<d)$ is the number of pairs with $d_{ij}<d$. To obtain unbiased estimate for $C(d)$, a minor correction must be added to compensate for the fact that the data originate from a finite time interval and not from an infinite time axis. (A correction of this kind is the standard one in the procedure of estimation of correlation function.) For self-similar data, C(d) must follow the power law:

$$C(d) \propto d^{D_c}$$

The D_c exponent here is the correlation dimension for a set of points on a line. Values $D_c < 1$ are indicative of self-similar clustering. The Poisson process is the boundary non-fractal case when $C(d) \propto d$, and $D_c = 1$. To find the empirical estimate for D_c, we can fit the empirical $C(d)$ function by a power law.

On Figures 4B and 4C, the empirical relationship $C(d)$ is plotted in the log-log scale. The linear fit over the entire delay range d = 10-10000 years yields an estimate of correlation dimension D_c =0.98 ≈ 1, indicating the absence of self-similar behavior (Fig. 4B). But within the restricted delay range 25-800 years, the slope of the approximating line equals 0.71 (Fig. 4C); and the null hypothesis $D_c = 1$ (the Poisson process) is rejected at the level of a statistical significance Q =2.5 %. One can conclude that in this specific delay range, the self-similar clustering is present, and it can be described by the a value of correlation dimension about 0.7.

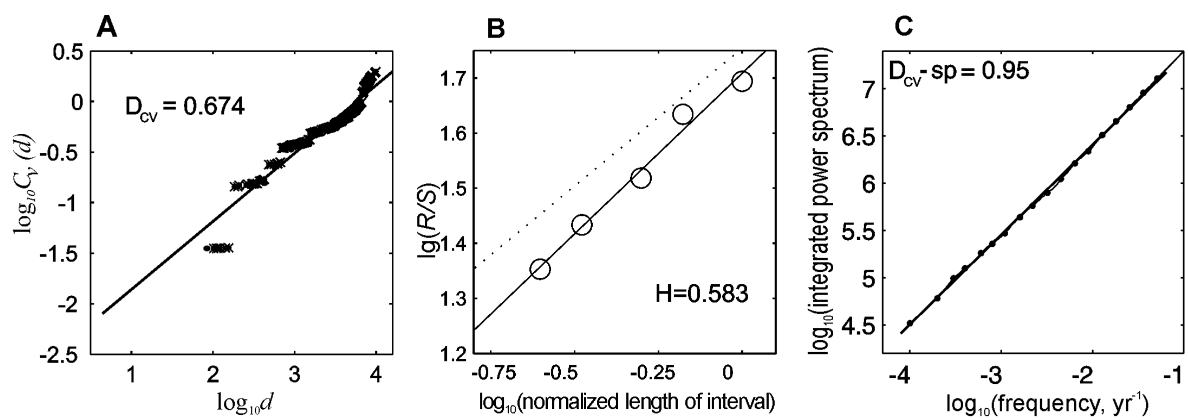

Figure 5. Plots illustrating self-similar properties of volcanic product output rate function. A: correlation integral for “weighted” points vs. delay d, yr; B: determination of the exponent H of the Hurst’s R/S method, for the range of time intervals 2500-10000 years (relative interval length from 0.25 to 1). The expected trend for the non-fractal case, with H=0.5, is shown by the dotted line. C: integrated power spectrum of the volcanic product output rate function, versus frequency. Good linear approximation of the data points indicates the acceptability of the hypothesis of the power law behavior of the power spectrum. The difference of the slope of this line from unity indicates the self-similar behavior of the process.

In addition to clustering of dates, examining of Figure 3 suggests that the largest events additionally "attract" one another. To test this idea, the same method of correlation integral (in the relevant modification) was applied to the temporal structure of the product output rate of volcanic eruptions, that is to the sequence of variable-weight points, with the weight equal to the volume V_i of products of a given eruption (Fig. 5A). The modified correlation integral $C_v(d)$ now looks as

$$C_v(d) = \frac{W(d)}{W(\infty)} = \frac{\sum_{d_{ij}<d} w(d_{ij})}{\sum_{\text{all } d_{ij}} w(d_{ij})}$$

where the cumulative number of pairs $N(d<d_{ij})$ is substituted by the cumulative number of weights $W(d)$, and the contribution into $W(d)$ from each event pair (i, j) with volumes V_i and V_j equals $w(d_{ij}) = V_iV_j$. The estimate of correlation dimension obtained in this way we denote D_{cv}. In this case, self-similar clustering can be revealed over the entire range of delays (excluding only smallest, inaccurately determined delays), and the estimate of D_{cv} equals 0.67. The null hypothesis "D_{cv} =1", (clustering of volumes is lacking) is rejected at the level of statistical significance Q = 3.4 %. This result is indicative of the presence of self-similar (fractal) temporal structure.

This structure is generated by a combined operation of two different, discernible phenomena. The first of them is the above-discussed common clustering of dates of events. The second phenomenon is the specific property of a temporal sequence of any variable-weight points that we call "ordinal clustering ". This kind of clustering is a property of the time-ordered list of eruptions (regardless to concrete dates), it is exhibited as the tendency of the largest eruptions (in contrast to smaller ones) to be the close neighbors in this list. The significance level for the hypothesis of the presence of ordinal clustering equals Q = 4.8 %.

Rescaled range and spectral approaches

In order to reveal fractal behavior in the data on volcanic product output rate (sequence of volumes), we also employed another popular technique, namely the rescaled range (R/S) method of Hurst (Fig. 5B). In our case this method is applicable only for the larger-delay range. The estimated value of the Hurst exponent is H = 0.58. It exceeds the value H = 0.5 for a non-fractal case, and the hypothesis "H>0.5" has the level of a statistical significance Q = 3,2 %. The presence of the contribution of ordinal clustering in forming the value H = 0.58 > 0,5 has the level of a statistical significance Q = 3.1 %.

We estimated correlation dimension also by the spectral method. On Figure 5C, we plotted the integral over frequency for the estimate of power spectrum (that is, for the sum of 29 Dirac delta functions with weights equal to volumes of eruptions). For a white noise or Poisson process, the integral should behave as f^1, and any significant periodic tendency must be seen as a bump on the graph. As can be seen from the actual graph, the integrated spectrum is smooth, close to a power law, and grows as $f^{0.95}$. In the "colored-noise" terms, the power spectrum of the volcanic product output rate is "pinkish", with a slight but clear tendency to truly pink or flicker noise (whose integrated spectrum grows slowly, as $\log_e(f)$). The value of the exponent (α=0.95) is an estimate of correlation dimension by the spectral method. For the hypothesis "α<1", the significance level is equal to Q= 6 %. For the hypothesis that ordinal clustering has made a contribution to this difference, the significance level is equal to Q = 4 %.

Discussion

Therefore, the reality of self-similar clustering or episodic behavior for the volcanic product output rate function is shown by three independent approaches. As for the value of correlation dimension D_{cv} parameter, one can note the difference between estimates obtained by correlation integral method (0.67) and by spectral method (0.95). The possibility of such a discrepancy is related to the fact that in the correlation integral approach, the contribution of large delays is dominating, whereas in the spectral approach, the dominating contribution is from high frequencies and thus of smaller delays. We do not consider this discrepancy as disturbing. A good match is obligatory only for really large samples, not for as small sequence size as 29.

The successful test of the presence of ordinal clustering by three different methods suggests the reality of this unusual phenomenon. A limited manifestation of common clustering when analyzed separately means that the phenomenon of ordinal clustering makes the crucial contribution to the self-similar behavior of the volcanic product output rate.

Both common clustering (for dates of events in time) and ordinal clustering (the tendency to enhanced proximity of largest events in the time-ordered event list) can be considered as two different manifestations of episodic behavior of volcanic process. And our results indicate more than mere presence of episodic tendency at some qualitative level. We observe self-similar, multiscale, fractal irregularity of volcanic product output rate for a territory of a size of some hundreds km, and for temporal scales 100-10000 years. This conclusion is in agreement with other observations of episodic behavior of volcanism, including both tendencies revealed in informal manner and direct indications of self-similar behavior. This last style was reported for a very wide range of temporal scales: hours, years and millions of years, and now is shown for hundreds to thousands of years. We believe that the self-similar episodic behavior (at difference with a Poisssonian one) represents a typical property of a sequence of eruptions within a volcanic area.

From the point of view of the nature of the phenomenon, the multiscale episodicity of volcanic product rate from a volcanic area may be caused by some external forcing or perturbing factor or factors with a wide range of characteristic times. A list of such factors may include: (1) variations of glacial load; (2) variations of an elastic stress field (formed by earthquakes and aseismic irreversible Earth strains); (3) variations of relevant parameters of subduction, including (3a) relative plate velocity, (3b) the volume of sediments at the particular patch of the plate being subducted and (3c) the amount of bound water in oceanic lithosphere at the same patch; and also (4) the effects of a unsteady flow of fluid and/or of a silicate liquid from deep mantle. Alternatively, one may ascribe the self-similar behavior to a certain intrinsic spatially-temporal dynamic structure of deep “volcanic root” processes. Like turbulence or seismicity, this hypothetic structure can have multiscale, fractal character and generate a wide spectrum of characteristic times "on its own", irrespective of any external forcing.

Conclusion

By means of three different statistical techniques, a self-similar episodic or clustering behavior is revealed for the temporal structure of the Holocene volcanic activity on Kamchatka, or, more precisely, for the volcanic product rate function. A very significant cause of episodic behavior is the ordinal clustering, that is the tendency of the largest eruptions to be unusually close neighbors in the time-ordered event list.

For more detailed exposition of a part of the presented results and discussion see: (Gusev and others, 2003). The research was supported by Russian Foundation for Basic Research through grants 03-05-64459 and 03-05-64027.

References

Cambray H. and Cadet J.-P., 1996, Synchronisme de l'activite volcanique d'arc: mythe ou realite? C.R. Acad.Sci.Paris, 322, ser IIa: 237-244

Chouet, B., and Shaw, H.R., 1991,. Fractal properties of tremor and gas piston events observed at Kilauea Volcano Hawaii. J. Geophys. Res. 96: 10177-10189.

DuBois, J., and Cheminee, J.-L., 1991,. Fractal analysis of eriptive activity of some basaltic volcanoes. J. Volcanol. Geotherm. Res. 45: 197-208 .

Godano,C. and L. Civetta., 1996,. Multifractal analysis of Vesuvius volcano eruptions. Geophys. Res. Lett. 23: 1167-1170.

Gusev A. A. Ponomareva V. V., Braitseva O. A., Melekestsev I. V., and Sulerzhitsky L. D., 2003, Great explosive eruptions on Kamchatka during the last 10,000 years: Self-similar irregularity of the output of volcanic products. J.Geophys.Res., 108: 2126

Ho C.-H., Smith E.I., Feuerbach D. L. and Naumann T.R.,, 1991,. Eruptive probability calculation for the Yucca Mountain site,USA: satisytical estimation of recurrence rates. Bull. Volcanol. 54: 50-56

Jones G., Chester D.K., Shooshtarian F., 1999, Statistical analyses of the frequency of eruptions at Furnas Volcano, Sao Miguel, Azores. Journ. Volcanol. Geotherm. Res. 92: 31-38

Kennett J.P., McBirney A.R., Thunell R.C., 1977, Episodes of Cenozoic volcanism in the circum-Pacific region. Journ. Volcanol. Geotherm. Res. 2: 145-163

Makarenko G.F., 1982,. Bursts of trap volcanism in Mesozoic and Cenozoic. Vulcanol. Seismol. No 4, 65-77

Pelletier J.D., 1999, Statistical self-similarity of magmatism and volcanism. J. Geophys. Res., 104: 15425-15438

Prueher L.M., Rea D.K., 2001,. Tephrochronology of the Kamchatka-Kurile and Aleutian arcs: evidence for volcanic episidicity. J Volcan. Geotherm Res 106: 67-87.

Rea D.K. and Scheidegger K.F., 1979, Eastern Pacific spreading rate fluctuation and its relation to Pacific area volcanic episodes. J. Volcanol Geotherm Res. 5, 135-148

Sigurdsson H. ed., 2000, Encyclopedia of Volcanoes, Academic Press, San-Diego, 1417p

Wickman, F.E., 1966, Repose patterns of volcanoes. I.Volcanic eruptions regarded as random phenomena. Ark Mineral Geol 4: 291-301